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Aggregation Operators of Fuzzy Hypersoft Sets

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Abstract

Multi criteria decision making (MCDM) is concerned about organizing and taking care of choice and planning issues including multi criteria. To solve these MCDM problems a new technique named as Fuzzy Hypersoft set is used to solve the uncertainty of MCDM. This paper includes basics of Fuzzy Hypersoft set and its operations like union, intersection, compliment etc., with the assistance of examples.
Keywords: Fuzzy Sets, Hypersoft Set, Fuzzy Hypersoft Sets, Aggregation Operators, Fundamentals

1. Introduction

In 1965, Zadeh [1] was the first who presented the concept of Fuzzy sets. In order to deal with various unpredictable situations and frequent real life issues Fuzzy Sets are used. Belongingness values are the basis of standard fuzzy sets. Sometimes it is difficult to assign the belongingness value for Fuzzy sets. In order to find the indefiniteness value the concept of interval valued fuzzy sets was presented [2].For suitable interpretation of an object in many unpredictable conditions we assume membership value as non-membership value. In this situation both the Fuzzy sets and IVF sets are not usable. Atanassov presented the idea of Intuitionistic Fuzzy sets to deal with such situation [3].Since IFS use both membership and non-membership values to work on ineffective data. IFS does not work on inexplicit and inconsistent data.

Another concept of Neutrosophic sets was given by Smarandache [4] to deal with indefinite and inconsistent data. Basically, a Neutrosophic set consists of truthiness, indeterminacy and falseness. This idea is useful for many purposes as these parameters are

independent to each other. Molodstov [5] was the first person who present the concept of Soft Sets to deal with various undefined and undetermined issues. Soft sets are used in various decisive issues.

Many scholars studied the fundamentals of Soft set theory in previous years. Maji et al.[7] studied many concepts including subset and superset and various operations between sets such as AND-OR product, restricted union and intersection was presented by Ali et al.[8].Soft matrix theory (SMT) was presented by Enginoglu and Cagman [10,9].The idea that a soft matrix is representative of a soft set was offered by Onyeozili and Singh[11].

Maji [13] presented that NS sets can be described by truthiness, indeterminacy and falseness values. To work on undefined and ineffective data NS sets are used whereas IFS and FSS can be used to work on restricted data. Smarandache [14] came up with a new method to work on indefinite data. He simplified the soft sets to hyper soft set. He examined the extensions of NS sets in MCDM and TOPSIS [16, 15, 17, 19, 18, and 20].

Decisive issues can be made easy by using various methods. In order to select the best option among various. We set various traits and options according to our problem and then by comparing with this set we can easily overcome these issues. This paper is a struggle a new technique called “FUZZY HYPERSOFT SET”.

2. Preliminaries

This section describes some basic definitions of soft set, Fuzzy soft set, Hyper soft set, Fuzzy Hyper soft set and operations like union, intersection, AND and OR product of (FHSS)

Definition 2.1: Soft Set:

If $\mathcal{N}$ be a universal set and set of parameters and attributes are represented by É$\boldsymbol{\acute{E}}$. Then the power set of $\mathcal{N}$ is denoted by $P(\mathcal{N})$ further $\mathcal{A} \sqsubseteq$ É$\boldsymbol{\acute{E}}$. Then a pair $(\mathcal{F}, \mathcal{A})$ is known as a soft set over $\mathcal{N}$ and it is represented by corresponding mapping:

$$\mathcal{F}: \mathcal{A} \longrightarrow P(\mathcal{N})$$

It can also be described as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(\boldsymbol{e}) \in P(\mathcal{N}) \textbf{such that } \mathbf{e} \in \mathcal{N}, \mathcal{F}(\mathbf{e}) = \boldsymbol{\phi} \ \boldsymbol{if} \ \boldsymbol{e} \neq \mathcal{A}\}$$

Definition 2.2: Fuzzy soft set:

If $\mathcal{N}$ be a universal set and the set of parameters or attributes with respect to $\mathcal{N}$ is represented by É. Power set of $\mathcal{N}$ is denoted by $\boldsymbol{P}(\mathcal{N})$ further $\mathcal{A} \sqsubseteq$ É. A Then a pair $(\mathcal{F}, \mathcal{A})$ is known as a fuzzy soft set over $\mathcal{N}$ and it is represented by a mapping given below:

$$\mathcal{F}: \mathcal{A} \longrightarrow \boldsymbol{P}(\mathcal{N})$$

Definition 2.3: Hyper Soft Set:

If $\mathcal{N}$ be a universal set and the power set of $\mathcal{N}$ is represented by $\mathrm{P}(\mathcal{N})$.Suppose that $\mathcal{k}^1, \mathcal{k}^2, \mathcal{k}^3 \ldots \mathcal{k}^n$,$\forall n \geq 1$is the set of n distinct parameters or attributes, where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{i} \cap \mathcal{K}^{j} = \emptyset$, for $i \neq j$ & $i, j \in \{1,2,3 \ldots n\}$ . .Therfore, this set $(\mathcal{F}, \mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n)$ is known as hypersoft set with respect to the universal set $\mathcal{N}$.

$$\mathcal{F}: \mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n \rightarrow \mathrm{P}(\mathcal{N})$$

3. Calculations

Definition 3.1 Fuzzy hyper soft Set (FHSS):

If $\mathcal{N}$ be a universal set and the power set of $\mathcal{N}$ is represented by $\mathrm{P}(\mathcal{N})$.Suppose that $\mathcal{k}^1, \mathcal{k}^2, \mathcal{k}^3 \ldots \mathcal{k}^n$,$\forall n \geq 1$is the set of n distinct parameters or attributes, where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{i} \cap \mathcal{K}^{j} = \emptyset$, for $i \neq j$ & $i, j \in \{1,2,3 \ldots n\}$ and the relation between them is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta$.Then this set $(\mathcal{F}, \delta)$ is known as Fuzzy hyper soft set (FHSS) with respect to the universal set $\mathcal{N}$.

$\mathcal{F}: \mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta \rightarrow P(\mathcal{N})$ &

$\mathcal{F}(\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n) = \{< \mathfrak{X},\ \mathcal{T}\big(\mathcal{F}(\delta)\big) such\ that\ \mathfrak{X} \in \mathcal{N}\}$

where $\mathcal{T}$ represents value of truthiness and represented by following mapping:

$$\mathcal{T}: \mathcal{N} \rightarrow [0,1].$$

Also $\quad 0 \leq \mathcal{T}\left(\mathcal{F}(\delta)\right) \leq 1.$

Example 3.1:

In order to determine which mobile phone is best let $\mathcal{N}$ be a set of decision makers given below:

$$\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$$

And the set of attributes is defined as follows:

$$\mathfrak{s}^1 = Mobile\ type, \mathfrak{s}^2 = RAM, \mathfrak{s}^3 = Sim\ Card, \mathfrak{s}^4 = Resolution, \mathfrak{s}^5 = Camera,$$

$$\mathfrak{s}^6 = Battery\ Power$$

Their respective parameters are given below:

$\mathcal{S}^1 = Mobile\ type = \{Qmobile, Vivo, Samsung, Iphone\}$

$\mathcal{S}^2 = RAM = \{6GB, 4\ GB, 2GB, 3GB\ \}$

$\mathcal{S}^3 = Sim\ Card = \{Single, Dual\}$

$\mathcal{S}^4 = Resolution = \{\ 2600 \times 4010 pixels, 1080 \times 780\ pixels, 1440 \times 3040\ pixels\}$

$$\mathcal{S}^5 = Camera = \{10\ MP, 12\ MP, 15\ MP\}$$

$$\mathcal{S}^6 = Battery\ Power = \{1000\ mAh, 4100\ mAh, 2050\ mAh\}$$

Let the function be $\mathcal{F}: \mathcal{S}^1 \times \mathcal{S}^2 \times \mathcal{S}^3 \times \mathcal{S}^4 \times \mathcal{S}^5 \times \mathcal{S}^6 \rightarrow P(\mathcal{N})$

Their Fuzzy values tables are given below:

$\mathcal{S}^1(Mobile\ type)$	ω^1	ω^2	ω^3	ω^4	ω^5
Q mobile	0.2	0.6	0.3	0.5	0.4
Vivo	0.6	0.2	0.2	0.7	0.4
Samsung	0.4	0.8	0.8	0.8	0.5
I phone	0.4	0.4	0.7	0.5	0.6

Table 1: Decision maker Fuzzy values for mobile type

$\mathcal{S}^2(RAM)$	ω^1	ω^2	ω^3	ω^4	ω^5
6 GB	0.2	0.3	0.4	0.4	0.2
4 GB	0.3	0.2	0.3	0.4	0.6
2 GB	0.6	0.8	0.7	0.5	0.4
3 GB	0.7	0.7	0.8	0.6	0.4

Table 2: Decision maker Fuzzy values for RAM

$\mathcal{S}^3(Sim\ Card)$	ω^1	ω^2	ω^3	ω^4	ω^5
Single	0.5	0.5	0.4	0.6	0.8
Dual	0.7	0.3	0.6	0.2	0.7

Table 3: Decision maker Fuzzy values for sim card

$\mathcal{S}^4(Resolution)$	ω^1	ω^2	ω^3	ω^4	ω^5
2600×4010	0.6	0.6	0.5	0.4	0.3
1080×780	0.2	0.6	0.7	0.5	0.2
1440×3040	0.4	0.5	0.4	0.8	0.6

Table 4: Decision maker Fuzzy values for resolution

$\mathcal{S}^5(Camera)$	ω^1	ω^2	ω^3	ω^4	ω^5
12 MP	0.5	0.6	0.5	0.3	0.8
10 MP	0.7	0.2	0.7	0.2	0.7
15 MP	0.4	0.4	0.7	0.6	0.6

Table 5: Decision maker Fuzzy values for camera

$\mathcal{S}^6(Battery\ Power)$	ω^1	ω^2	ω^3	ω^4	ω^5
1000 m Ah	0.6	0.6	0.3	0.8	0.4
4100 m Ah	0.2	0.2	0.2	0.7	0.4
2050 m Ah	0.4	0.9	0.8	0.6	0.5

Table 6: Decision maker Fuzzy values for battery power

Fuzzy hyper soft set (FHSS) can be described as:

$$\mathcal{F}: (\mathcal{S}^1 \times \mathcal{S}^2 \times \mathcal{S}^3 \times \mathcal{S}^4 \times \mathcal{S}^5 \times \mathcal{S}^6) \rightarrow P(\mathcal{N})$$

Let's assume that $\mathcal{F}(\delta) = \mathcal{F}(vivo, 2\ GB, Dual) = \{\omega^1, \omega^4\}$

Then (FHSS) of above supposed relation is

$$\mathcal{F}(\delta) = \mathcal{F}(Vivo, 2GB, Dual\ ) = \{< \omega^1, (Vivo\{0.7\}, 2GB\{0.7\}, Dual\{0.8,\}) > \\ < \omega^4 (vivo\{0.8\}, 2\ GB\{0.6\}, Dual\{0.3\}) >\}$$

Tabular form of above is given below:

$\mathcal{F}(\delta) = \mathcal{F}(vivo, 2\ GB, Dual\ )$	ω^1	ω^4
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.3

Table 7: Tabular form of Fuzzy Hyper soft Set (FHSS)

Definition 3.2: Fuzzy Hyper soft Subset:

Let two (FHSS) over the same universe$\mathcal{N}$ be $\mathcal{F}(\delta^1)$and $\mathcal{F}(\delta^2)$.Suppose that $\mathcal{k}^1, \mathcal{k}^2, \mathcal{k}^3 \ldots \mathcal{k}^n$ for $n \geq 1$, is the set of n distinct parameters or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{i} \cap \mathcal{K}^{j} = \emptyset$, for $i \neq j$ & $i, j \epsilon \{1,2,3 \ldots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta$.Then $\mathcal{F}(\delta^1)$is the Fuzzy hyper soft subset of $\mathcal{F}(\delta^2)$ when

$$\mathcal{T}(\mathcal{F}(\delta^1)) \leq \mathcal{T}(\mathcal{F}(\delta^2)\ )$$

Mathematical example of (FHSS):

Let $\mathcal{F}(\delta^1)$and$\mathcal{F}(\delta^2)$ be the two FHSS with respect to the same universe $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. The FHSS $\mathcal{F}(\delta) = \mathcal{F}(vivo, 2\ GB, Dual\ ) = \{\omega^1, \omega^4\}$ is the subset of FHSS $\mathcal{F}(\delta^2) = \mathcal{F}(Vivo, 2GB) = \{\omega^1\}$ if $\mathcal{T}(\mathcal{F}(\delta^1)) \leq \mathcal{T}(\mathcal{F}(\delta^2))$.

Tabular representation is given below:

$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual\ )$	ω^1	ω^4

Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 8: Tabular form of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

Table 9: Tabular form of FHSS $\mathcal{F}(\delta^2)$

It can also be written as:

$$\mathcal{F}(\delta^1) \subset \mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ \ GB, Dual\ ) \subset \mathcal{F}(vivo, 2\ GB)$$

$$= \begin{Bmatrix} < \omega^1, (vivo\{0.6\}, 2\ GB\{0.6\}, Dual\{0.7\}) >, \\ < \omega^4 (vivo\{0.7\}, 2\ GB\{0.5\}, Dual\{0.2\}) > \end{Bmatrix}$$

$$\subset \{< \omega^1, (vivo\{0.8\}, 2\ GB\{0.7\})>\}$$

As by the definition of Fuzzy hyper soft subset $\mathcal{T}(\mathcal{F}(\delta^1)) \leq \mathcal{T}(\mathcal{F}(\delta^2))$ and the associated value of vivo for ω^1 in both sets are 0.6 and 0.8 which satisfy the definition of Fuzzy hyper soft subset as $0.6 < 0.8$.This shows that $0.6 \subset 0.8$ and the other attributes of FHSS $\mathcal{F}(\delta^1)$ and FHSS $\mathcal{F}(\delta^2)$ will also satisfy this condition.

Definition 3.3: Fuzzy equal hyper soft Set:

If $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ be two (FHSS) set over same universe $\mathcal{N}$. Suppose that $k^1, k^2, k^3 \dots k^n$ for $n \geq 1$, is the set of n distinct parameter or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \dots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^i \cap \mathcal{K}^j = \emptyset$, for $i \neq j$ & $i, j \epsilon \{1,2,3 \dots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \dots \mathcal{K}^n = \delta$ .Then $\mathcal{F}(\delta^1)$ is the Fuzzy equal hyper soft subset of $\mathcal{F}(\delta^2)$ if:

$$\mathcal{T}\big(\mathcal{F}(\delta^1)\big) = \mathcal{T}\big(\mathcal{F}(\delta^2)\big)$$

Mathematical Example of equal Fuzzy hyper soft Set:

Let two (FHSS) are $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ over the same universe $\mathcal{N}\{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$.The FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual) = \{\omega^1, \omega^4\}$ is the equal to FHSS

$\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2GB) = \{\omega^1\}$ when $\mathcal{T}(\mathcal{F}(\delta^1)) = \mathcal{T}(\mathcal{F}(\delta^2))$,

Tabular form of above is given below:

$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual)$	ω^1	ω^4
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 10: Tabular form of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ GB)$	ω^1
Vivo	0.6
2 GB	0.6

Table 11: Tabular form of FHSS $\mathcal{F}(\delta^2)$

It can also be written in the form:

$$\begin{aligned}\left(\mathcal{F}(\delta^1) = \mathcal{F}(\delta^2)\right) &= \left(\mathcal{F}(vivo, 2\ GB, Dual) = \mathcal{F}(vivo, 2GB)\right)\\ &= \left((\{< \omega^1, (vivo\{0.6\}, 2GB\{0.6\}, Dual\{0.7\}) >,\right.\\ &< \omega^4(vivo\{0.7\}, 2\ GB\{0.5\}, Dual\{0.2\}) >\}\\ &\left.= \{< \omega^1, (vivo\{0.6\}, 2\ GB\{0.6\}) >\})\right)\end{aligned}$$

As by the definition of equal fuzzy hyper soft set $\mathcal{T}(\mathcal{F}(\delta^1)) = \mathcal{T}(\mathcal{F}(\delta^2))$, here the membership value of vivo for ω^1 in both sets is 0.6 and 0.6 which satisfy the condition for equal fuzzy hyper soft $0.6 = 0.6$ .This shows that $\mathcal{T}(\mathcal{F}(\delta^1)) = \mathcal{T}(\mathcal{F}(\delta^2))$ and the other attributes of FHSS $\mathcal{F}(\delta^1)$ and FHSS $\mathcal{F}(\delta^2)$ will also satisfy this condition.

Definition 3.4: Null Fuzzy hyper soft Set:

Consider $\mathcal{F}(\delta^1)$be a (FHSS) with respect to a universal set $\mathcal{N}$. Suppose that $\mathscr{k}^1, \mathscr{k}^2, \mathscr{k}^3 \dots \mathscr{k}^n$ for $n \geq 1$, is the set of n distinct parameter or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \dots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{\imath} \cap \mathcal{K}^{\jmath} = \emptyset$, for $\imath \neq \jmath$ & $\imath, \jmath \epsilon \{1,2,3 \dots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \dots \mathcal{K}^n = \delta$. Then $\mathcal{F}(\delta^1)$ is known as Null Fuzzy hypersoft set if

$$\mathcal{T}\left(\mathcal{F}(\delta^1)\right) = 0$$

Mathematical example of Null (FHSS):

Let a Fuzzy hypersoft set be $\mathcal{F}(\delta^1)$ over the universe $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$.The FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual) = \{\omega^1, \omega^4\}$ is known as null FHSS if its Fuzzy values are 0. Tabular representation of null FHSS is given below:

$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual)$	ω^1	ω^4
Vivo	0	0
2 GB	0	0
Dual	0	0

Table 12: Tabular form of FHSS $\mathcal{F}(\delta^1)$

It can also be written as

$$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual)$$
$$= \{< \omega^1, (vivo\{0\}, 2\ GB\{0\}, Dual\{0\}) >,$$
$$< \omega^4(vivo\{0\}, 2GB\{0\}, Dual\{0\}) >\}$$

Definition 3.5: Compliment of Fuzzy hypersoft Set

Suppose that $\mathcal{F}(\delta^1)$ be a Fuzzy hypersoft set with respect to a universal set $\mathcal{N}$. Suppose that $\mathscr{k}^1, \mathscr{k}^2, \mathscr{k}^3 \dots \mathscr{k}^n$ for $n \geq 1$, is the set of n distinct parameters or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \dots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{\imath} \cap \mathcal{K}^{\jmath} = \emptyset$, for $\imath \neq \jmath$ & $\imath, \jmath \epsilon \{1,2,3 \dots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \dots \mathcal{K}^n = \delta$. Then the compliment of $\mathcal{F}(\delta^1)$ is denoted as $\mathcal{F}^c(\delta^1)$ represented by following mapping:

$$\mathcal{F}^c(\delta^1): (\neg \mathcal{K}^1 \times \neg \mathcal{K}^2 \times \neg \mathcal{K}^3 \dots \neg \mathcal{K}^n) \to P(\mathcal{N})$$

$\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

If

$$\mathcal{T}^C(\mathcal{F}(\delta^1)) = 1 - \mathcal{T}^C(\mathcal{F}(\delta^1))$$

Mathematical example of compliment of FHSS:

Suppose a Fuzzy hypersoft set $\mathcal{F}(\delta^1)$ with respect to a universal set $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. The compliment of FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual) = \{\omega^1, \omega^4\}$ is given as $\mathcal{T}^C(\mathcal{F}(\delta^1)) = 1 - \mathcal{T}^C(\mathcal{F}(\delta^1))$

Its tabular form is given below

$\mathcal{F}^C(\delta^1) = \mathcal{F}(Not\ vivo, Not\ 2\ GB, Not\ \ dual)$	ω^1	ω^4
Not vivo	0.4	0.3
Not 2 GB	0.4	0.5
Not Dual	0.3	0.8

Table 13: Tabular form of NHSS $\mathcal{F}(\delta^1)$

This can also be written as

$$\mathcal{F}^C(\delta^1) = \mathcal{F}(\ not\ vivo, not\ 2\ GB, not\ Dual\ )$$
$$= \{< \omega^1, (not\ vivo\{0.4\}, not\ 2GB\{0.4\}, not\ Dual\{0.3\}) >,$$
$$< \omega^4(not\ vivo\{0.3\}, not\ 2\ GB\{0.5\}, not\ Dual\{0.8\}) >\}$$

Here we can see that membership value of vivo for ω^1 in $\mathcal{F}(\delta^1)$is 0.6 and its compliment is 0.4 which satisfy the definition of compliment of (FHSS). This shows that 0.4 is the compliment of 0.6 and same was the case with the rest of the attributes of FHSS $\mathcal{F}(\delta^1)$.

Definition 3.6: Union of Two Fuzzy Hypersoft Set:

Consider $\mathcal{F}(\delta^1)$ and$\mathcal{F}(\delta^2)$ be two Fuzzy hypersoft set over common universe $\mathcal{N}$. Suppose that $\mathcal{k}^1, \mathcal{k}^2, \mathcal{k}^3 \ldots \mathcal{k}^n$ for $n \geq 1$, is the set of n distinct parameter or attributes where the set

$\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^i \cap \mathcal{K}^j = \emptyset$, for $i \neq j$ & $i, j \epsilon \{1,2,3 \ldots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta$.Then $\mathcal{F}(\delta^1) \cup \mathcal{F}(\delta^2)$is given as

$$\mathcal{T}\left(\mathcal{F}(\delta^1) \cup \mathcal{F}(\delta^2)\right) = \begin{cases} \mathcal{T}\left(\mathcal{F}(\delta^1)\right) & if\ \mathfrak{X} \in \delta^1 \\ \mathcal{T}\left(\mathcal{F}(\delta^2)\right) & if\ \mathfrak{X} \in \delta^2 \\ s\left(\mathcal{T}\left(\mathcal{F}(\delta^1)\right), \mathcal{T}\left(\mathcal{F}(\delta^2)\right)\right) & if\ \mathfrak{X} \in \delta^1 \cap \delta^2 \end{cases}$$

Mathematical example of Fuzzy hypersoft Set Union:

The two FHSS with respect to a universal set $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$ are $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ .Tabular form of FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual\ ) = \{\omega^1, \omega^4\}$ and FHSS $\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ GB) = \{\omega^1\}$ is given as:

$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2\ GB, Dual\ )$	ω^1	ω^4
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 14: Tabular form of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(\text{Vivo}, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

Table 15: Tabular Representation of FHSS $\mathcal{F}(\delta^2)$

Then the union of above FHSS is given as

$\mathcal{F}(\delta^1) \cup \mathcal{F}(\delta^2)$	ω^1	ω^4
Vivo	0.8	0.7
2 GB	0.7	0.5
Dual	0.7	0.2

Table 16: Table of Union of FHSS $\mathcal{F}(\delta^1)$ and FHSS $\mathcal{F}(\delta^2)$

It can also be written in the following form:

$$\begin{aligned}\mathcal{F}(\delta^1) \cup \mathcal{F}(\delta^2) &= \mathcal{F}(vivo, 2\ GB, Dual\ ) \cup \mathcal{F}(vivo, 2\ GB)\\ &= \{< \omega^1, (vivo\{0.8\}, 2GB\{0.7\}, Dual\{0.7\}) >,\\ &< \omega^4(vivo\{0.7\}, 2\ GB\{0.5\}, Dual\{0.2\}) >\}\end{aligned}$$

Definition 3.7: Intersection of Two Fuzzy hypersoft Set:

Consider the two FHSS $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ over the same universe $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. Suppose that $k^1, k^2, k^3 \ldots k^n$ for $n \geq 1$, is the set of n distinct parameters or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^i \cap \mathcal{K}^j = \emptyset$, for $i \neq j$ & $i, j \epsilon \{1,2,3 \ldots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta$ .So the intersection of two FHSS $\mathcal{F}(\delta^1) \cap \mathcal{F}(\delta^2)$ is given below:

$$\mathcal{T}\big(\mathcal{F}(\delta^1) \cap \mathcal{F}(\delta^2)\big) = \begin{cases} \mathcal{T}\big(\mathcal{F}(\delta^1)\big) & if\ \ \mathfrak{X} \in \delta^1 \\ \mathcal{T}\big(\mathcal{F}(\delta^2)\big) & if\ \mathfrak{X} \in \delta^2 \\ \mathrm{t}\Big(\mathcal{T}\big(\mathcal{F}(\delta^1)\big), \mathcal{T}\big(\mathcal{F}(\delta^2)\big)\Big) & if\ \ \mathfrak{X} \in \delta^1 \cap \delta^2 \end{cases}$$

Mathematical example of Fuzzy hypersoft Set Intersection

Consider the two FHSS $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ over the same $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$.Tabular representation of FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2GB, Dual\ ) = \{\omega^1, \omega^4\}$ and NHSS $\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2GB) = \{\omega^1\}$ is given below

$\mathcal{F}(\delta^1) = \mathcal{F}(vivo, 2GB, Dual\ )$	ω^1	ω^4
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 17: Tabular Representation of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(vivo, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

Table 18: Tabular Representation of FHSS $\mathcal{F}(\delta^2)$

Then the intersection of above FHSS is given as

$\mathcal{F}(\delta^1) \cap \mathcal{F}(\delta^2)$	ω^1
Vivo	0.6
2 GB	0.6
Dual	0.0

Table 19: Intersection of FHSS $\mathcal{F}(\delta^1)$ and FHSS $\mathcal{F}(\delta^2)$

It can also be written as:

$$\mathcal{F}(\delta^1) \cap \mathcal{F}(\delta^2) = \mathcal{F}(\text{vivo}, 2GB, Dual\ ) \cap \mathcal{F}(\text{vivo}, 2\ GB)$$
$$= \{< \omega^1, (\text{vivo}\{0.6\}, 6\ GB\{0.6\}, Dual\{0.0\}) >\}$$

Definition 3.8: AND Operation on two Fuzzy hypersoft Set:

Consider the two FHSS $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$ over the same universe $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. Suppose that $\mathscr{k}^1, \mathscr{k}^2, \mathscr{k}^3 \ldots \mathscr{k}^n$ for $n \geq 1$, is the set of n distinct parameter or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \ldots \mathcal{K}^n$ is the respective values of attributes with $\mathcal{K}^{\imath} \cap \mathcal{K}^{\jmath} = \emptyset$, for $\imath \neq \jmath$ & $\imath, \jmath \epsilon \{1,2,3 \ldots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \ldots \mathcal{K}^n = \delta$. Then $\mathcal{F}(\delta^1) \wedge \mathcal{F}(\delta^2) = \mathcal{F}(\delta^1 \times \delta^2)$ is given as:

$$\mathcal{T}(\delta^1 \times \delta^2) = t\left(\mathcal{T}\left(\mathcal{F}(\delta^1)\right), \mathcal{T}\left(\mathcal{F}(\delta^2)\right)\right)$$

Numerical Example of AND-Operation

Let two FHSS are $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$with respect to a universal set $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. Tabular form of FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(\text{vivo}, 2\ GB, Dual\ ) = \{\omega^1, \omega^4\}$ and FHSS $\mathcal{F}(\delta^2) = \mathcal{F}(\text{vivo}, 2\ GB,) = \{\omega^1\}$ is given as:

$\boldsymbol{\mathcal{F}(\delta^1)}$ $= \boldsymbol{\mathcal{F}(\text{vivo}, 2\ GB, Dual\ )}$	$\boldsymbol{\omega^1}$	$\boldsymbol{\omega^4}$
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 20: Tabular representation of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(\text{vivo}, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

Table 21: Tabular representation of FHSS $\mathcal{F}(\delta^2)$

Then the AND Operation of above FHSS is given as

$\mathcal{F}(\delta^1) \wedge \mathcal{F}(\delta^2)$	ω^1	ω^4
vivo $\times$ vivo	0.6	0.0
vivo $\times\ 2\ GB$	0.6	0.0
$2\ GB\ \times$ vivo	0.6	0.0
$2\ GB \times\ 6\ GB$	0.6	0.0
$Dual\ \times$ vivo	0.7	0.0
$Dual \times\ 2\ GB$	0.7	0.0

Table 22: Tabular representation of AND of FHSS $\mathcal{F}(\delta^1) \wedge \mathcal{F}(\delta^2)$

Definition 3.9: OR Operation on Two Fuzzy Hypersoft Set

Let two FHSS are $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$with respect to a universal set $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$.. Suppose that $\mathcal{k}^1, \mathcal{k}^2, \mathcal{k}^3 \dots \mathcal{k}^n$ for $n \geq 1$, is the set of n distinct parameter or attributes where the set $\mathcal{K}^1, \mathcal{K}^2, \mathcal{K}^3 \dots \mathcal{K}^n$ is the consistent attributive values with $\mathcal{K}^i \cap \mathcal{K}^j = \emptyset$, for $i \neq j$ & $i, j \epsilon \{1,2,3 \dots n\}$ and the relation is $\mathcal{K}^1 \times \mathcal{K}^2 \times \mathcal{K}^3 \dots \mathcal{K}^n = \delta$. Then $\mathcal{F}(\delta^1) \vee \mathcal{F}(\delta^2) = \mathcal{F}(\delta^1 \times \delta^2)$ is given as

$$\mathcal{T}(\delta^1 \times \delta^2) = t\left(\mathcal{T}\left(\mathcal{F}(\delta^1)\right), \mathcal{T}\left(\mathcal{F}(\delta^2)\right)\right)$$

Mathematical example of OR-Operation

Let two FHSS are $\mathcal{F}(\delta^1)$ and $\mathcal{F}(\delta^2)$with respect to a universal set $\mathcal{N} = \{\omega^1, \omega^2, \omega^3, \omega^4, \omega^5\}$. Then representation of FHSS $\mathcal{F}(\delta^1) = \mathcal{F}(\text{vivo}, 2\ GB, Dual\ ) = \{\omega^1, \omega^4\}$ and FHSS $\mathcal{F}(\delta^2) = \mathcal{F}(\text{vivo}, 2\ GB,) = \{\omega^1\}$ is described below:

$\mathcal{F}(\delta^1) = \mathcal{F}(\text{vivo}, 2GB, Dual\ )$	ω^1	ω^4
Vivo	0.6	0.7
2 GB	0.6	0.5
Dual	0.7	0.2

Table 23: Tabular representation of FHSS $\mathcal{F}(\delta^1)$

$\mathcal{F}(\delta^2) = \mathcal{F}(\text{vivo}, 2\ GB)$	ω^1
Vivo	0.8
2 GB	0.7

Table 24: Tabular representation of FHSS $\mathcal{F}(\delta^2)$

Then the OR Operation of above FHSS is given as

$\mathcal{F}(\delta^1)\ \mathcal{F}(\delta^2)$	ω^1	ω^4
vivo $\times$ vivo	0.8	0.7
vivo $\times$ $2\ GB$	0.7	0.7
$2\ GB$ $\times$ vivo	0.8	0.5
$2\ GB \times 2\ GB$	0.7	0.5
$Dual$ $\times$ vivo	0.8	0.2
$Dual \times 2\ GB$	0.7	0.2

Table 25: Tabular representation of OR operation of FHSS $\mathcal{F}(\delta^1)$ and FHSS $\mathcal{F}(\delta^2)$

4. Conclusions

This paper explained Fuzzy Hypersoft set and various operations on FHSS such as union, intersection, Compliment, AND-OR product which are useful in many Multi Criteria Decision Making issues with the help of method used in [23].

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